The Design Of A Floating Cup Swash System

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Abstract

The Floating Cup principle offers many benefits over conventional hydraulic axial displacement machines. It allows for high efficiency, low noise levels and low starting torques at a competing price level. Constant displacement Floating Cup pumps have been build and tested thoroughly and the first Floating Cup pump with variable displacement has been presented already. It differs from conventional open circuit pumps in the fact that oil is fed through the swash plates, which vibrate constantly during operation. To secure the high efficiency, the sealing interface between barrel and swash plate must remain tight under all nominal working conditions. Here, a method for dynamical analysis is being presented that enables for the dedicated design of the Floating Cup swash system. The critical design parameters of the swash system are found by means of a linear model. From there, a parameter variation study is performed through numerical simulation in a non-linear model.

KEYWORDS: Axial piston pump, floating cup, open circuit, variable displacement, dynamics

Nomenclature

A_c	Actuator pressure area	m^2
C_d	Turbulent flow coefficient	-
D_i	Restriction opening diameter i=1,2	m
E_{oil}	Bulk modulus of oil	N/m^2
f_0	Natural eigenfrequency of the linearised swash system	rad/s
G(s)	Transfer function swash system	
I	Swash plate inertia with respect to swash axis	kgm^2
I_{barrel}	Barrel inertia with respect to swash axis	kgm^2
$M_{act,\ comp,\ swivel}$	Actuator, compensator, swivel torque respectively	Nm
p(t)	Actuator pressure	Pa
p_0	Atmospheric pressure	Pa
p_A	Pump pressure	Pa
p_T	Tank pressure	Pa
p_{SS}	Actuator pressure at steady state	Pa
Δp	Actuator pressure variation around p_{SS}	Pa
P(s)	Laplace transform of Δp	-
Q_j	Flow j=1,2,3	m^3/s
R	Actuator leverage	m
V_d	Dead volume actuator	m^3
V_0	Total volume actuator	m^3
x, x_d	Actuator piston position	m
α_b	Barrel angular acceleration around swash axis	rad/s^2
φ, φ_d	Swash plate angle	rad
ρ_{oil}	Oil density	kg/m^3
ω	Angular speed swash plate around swash axis	rad/s
Ω	Laplace transform of ω	-
ζ	Damping ratio of linearised swash system	-

1 Introduction

The recurring design specifications for hydraulic equipment are well known to the hydraulic industry. Manufacturers of positive displacement machines like the slipper type and the bent-axis are doing their utmost to reduce production costs, reduce noise levels and push machine efficiency towards the physical boundaries. By combining the benefits of slipper type and bent axis into one unit, and adding to that the know-how of modern cost effective production methods, the Floating Cup technology can open up new worlds for the hydraulic industry. A constant displacement unit showing an overall peak efficiency of 95% [2] and low noise levels at a competing price has already captured the industry's attention. The main focus though is on variable units, since that is the biggest part of the hydraulic market for pumps. Recently the open circuit Floating Cup variable pump has been presented [3].

The rotary group of a variable displacement Floating Cup pump is quite similar to that of a constant displacement unit [1]. The design is axially symmetrical and at both ends of the rotary group lies a port plate. The displacement volumes are made up of separate cylinders, which float on the barrel plate. Each cylinder or ' floating cup' is sealed up by a spherical piston. The pistons are rigidly mounted on the rotor, a flange-like radial extension of the axle. The port plates lie on top of a swash plate, which can be rotated around the swash axis by a set of actuators and one compensator. The swash plate determines the displacement of the machine by tilting the barrel plate with respect to the rotor. The combination of actuators, compensator and swash plate is referred to as the swash system, which is being highlighted in **figure 1**.

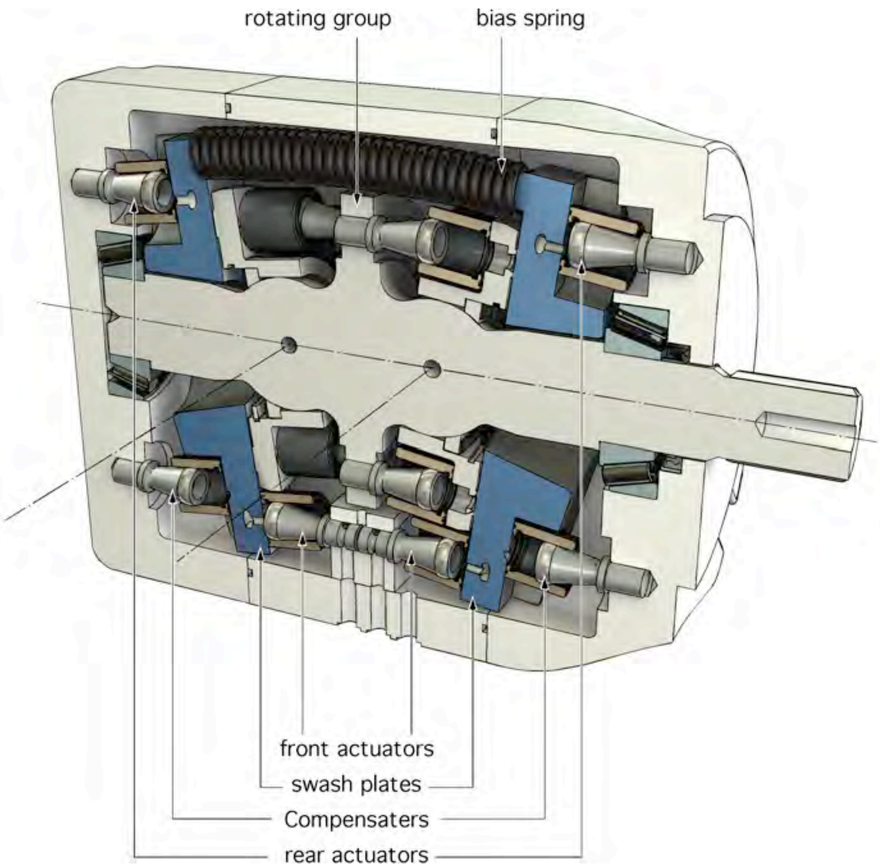


Figure 1 Cross section of the open circuit, variable displacement Floating Cup design

Contrary to slipper type pumps, the main oil flow in a variable Floating Cup pump is fed through the swash plate. To maintain a high efficiency, the gap between barrel and port plate must remain sufficiently small under all working conditions. A periodically varying torque applied by the commutating rotary group continuously excites the swash plates. The exciting torque, also known as 'swivel torque' [4] causes vibration of the swash plate and barrel, that could lead to high leakage flows. Therefore the barrel rotation around the swash axis must remain in phase with the swash plate motion.

What now are the design parameters that determine the dynamical behaviour of the swash system and what is necessary for the barrel to be able to follow its movements? What choice for the parameter values avoids system resonance, given a typical external periodic excitation? These are the questions that will be answered in this paper based on the analysis of a simplified swash system model.

2 Simple model of an axial piston pump swash system

The swash system of any variable axial piston pump can be reduced to a simple model containing a rotating mass of inertia controlled by an actuator piston. The actuator is fed with pressurized oil through a restriction with variable flow diameter. **Figure 2** shows this model graphically: the actuator is filled with oil at a pressure that varies in time. The angular position of the inertia, which stands for the swash plate angle, is also a time dependent model state.

Though the model is applicable to a wide range of swash systems, here the Floating Cup system will be discussed specifically. In the Floating Cup design there are two actuators, positioned in such a way that together they solely produce a torque around the swash axis. In the model these are combined into one actuator piston. The compensator piston is connected to high pump pressure, acting against the resulting force of the pumping pistons, creating a preloading torque around the swash axis.

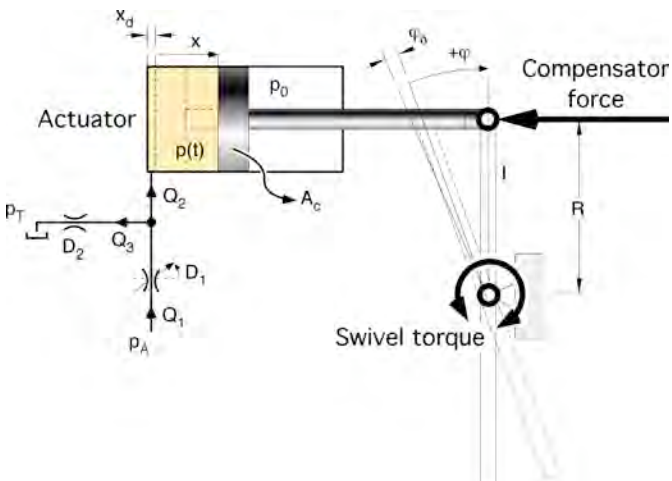


Figure 2 Simplified model of a generic swash system for any variable axial piston pump.

Thus, there are three torque loads acting on the swash system with respect to the swash axis: an actuator torque, a compensator torque and the previously mentioned swivel torque. For the dynamical analysis, the constant torque terms, which determine equilibrium, can be ignored. The variations in time around the steady state values are only of interest here.

3 Swash system design criterion with respect to barrel acceleration

When the swash plate moves around its axis, the barrel rotates around the same axis, driven by the swash plate. The force that drives the barrel and gives it the necessary angular acceleration to follow the swash plate is the interacting force, or contact force between swash plate and barrel (**figure 3**). It is constant or pressure dependent, depending on whether the barrel is hydraulically balanced or not. Given a certain pump pressure, say 450 bar, the barrel acceleration α_b is limited to a maximum:

$$\alpha_{b,max} = \frac{F_{contact} \cdot y_{max}}{I_{barrel}} \quad (1)$$

If the swash plate acceleration exceeds $\alpha_{b,max}$, the barrel and swash plate move out of phase, resulting in high leakage flows. Therefore it is important that the maximum acceleration of the swash system can be controlled through its design. The crucial design parameters that influence the swash system's dynamics have to be adapted to the barrel dimensions, inertia and contact force. It is understood that this is a simplified representation of a highly complex physical phenomenon. But for the first system explorations it will suffice.

When both restrictions in figure 2 have a small flow diameter, the spring characteristics of the oil trapped inside the actuator govern the system's dynamical behaviour. In other words, for small restriction diameters the swash system behaves like a damped mass-spring system. Resonance can occur if the excitation holds frequencies that coincide with the natural frequency of the system. If the system is brought into resonance, the acceleration of the inertia reaches its maximum amplitude. A linear analysis of the simple model helps to define the influencing design parameters.

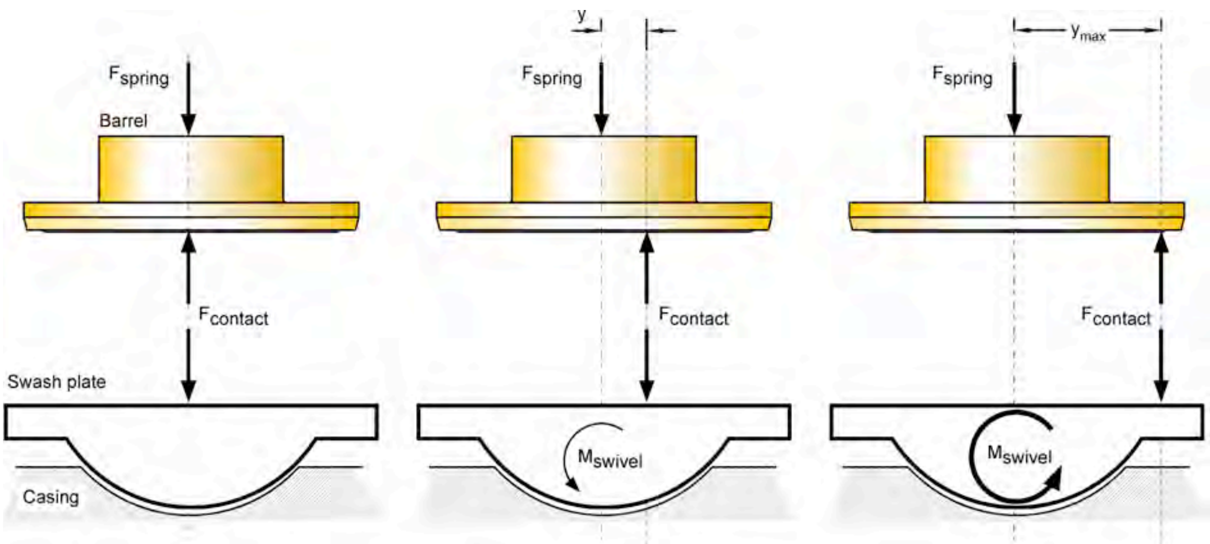


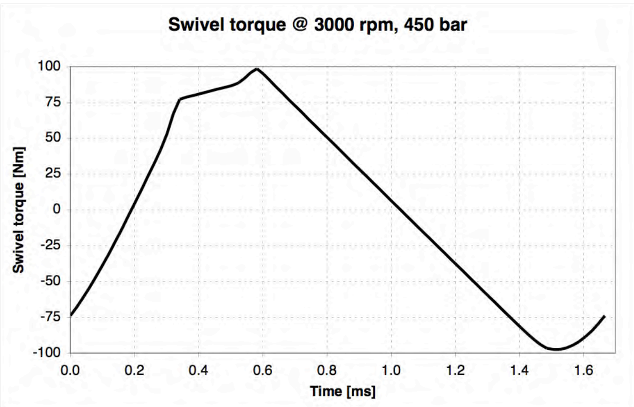
Figure 3 Simplified representation of the interaction between the barrel and the swash plate at three situations: M_{swivel} at zero, sub maximum and maximum magnitude. For simplicity, the port plate is treated as a part of the swash plate and is not depicted.

4 Finding the governing design parameters

The swash system shows non-linear behaviour for large pressure variations inside the actuator piston. To capture the design parameters that dominate the magnitude of the swash plate accelerations, a linearised model has been deduced. In the appendix the analytical procedure is laid out, here the physical interpretation is being addressed.

As a starting point, the swivel torque signal is taken from the simulation model of a constant displacement Floating Cup pump. At a given pump speed or ‘rotation frequency’, the swivel torque has a base frequency that is 12 times higher, corresponding to the number of pistons on one side. In analogy to the Fourier expansion of the swivel torque signal, the base frequency is called the first order content of the excitation. Superimposed to that are the second, third, fourth and so on. **Figure 4** shows the swivel torque in the time domain as well as the frequency content after Fourier expansion.

Now that the excitation has been defined, let us look at the dynamical system it excites. The swash system can be described in terms of stiffness, damping and inertia. Stiffness is the spring quality that comes from the actuators acting as oil springs. A given rotational deviation from steady state, results in a pressure rise inside the actuators, pushing the swash plate back towards the steady state position. It figures that enlarging the actuator area (A_c) increases system stiffness. The same goes for increasing the radial position (R) of the actuator with respect to the swash axis. The oil volume (V_0) itself also influences the oil spring stiffness. Enlarging the dead volume (V_d) of the actuators results in a decrease in stiffness.



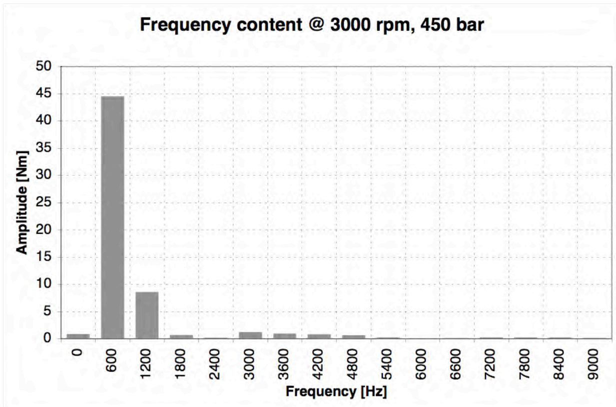


Figure 4 Swivel torque signal in the time domain, as well as the frequency domain. The time signal originates from an AMESim model of a constant displacement Floating Cup pump.

The damping phenomenon comes mainly from the restrictions that feed the swash system with oil. If the restrictions would be closed, the system would behave almost like a mass bouncing on an undamped spring. Opening the restrictions (D_1, D_2) gradually increases the energy dissipation through pressure throttling, creating an increasing damping ratio for the mass-spring system.

The inertia (I) of the swash plate also influences the system’s response to the excitation. A very light system reacts to a wide range of excitation frequencies. A heavy system only reacts to relatively low frequencies. Choosing the swash plate inertia large would secure the design criterion of maximum acceptable acceleration but would result in very long response times. Defining the optimum inertia is therefore a trade-off between efficiency and maximum swash speed.

Opening the restrictions further and further would in the end result in a system that hardly builds up any pressure inside the actuator during excitation, thus reducing the damping as well as the stiffness. In such a theoretical case, the swash plate acceleration would merely be depending on inertia and swivel torque magnitude. From the system gain deduced in the linear analysis (appendix), it follows that the restriction diameter cannot be used to minimize the swash acceleration under all working conditions. Depending on the excitation frequency, the optimal restriction opening is either zero or infinite. The restriction diameter is therefore set at 0.5 mm; a value for which the steady state flow into the tank equals 1.5 L/min.

Summarizing, the design parameters that govern the swash system's dynamics are actuator area, actuator leverage, actuator dead volume and swash plate inertia. The mathematical analysis of a linearised model, presented in the appendix, confirms this conclusion. The linear model is not sufficiently accurate for simulation of system responses. Therefore, the non-linear model from figure 2 has been implemented into the simulation software AMESim. The simulation model was used to perform a parameter variation. The simulation results are used, to visualise the correlation between changing parameter values and system behaviour.

5 Numerical parameter variation in a non-linear model

The parameter space spanned by the parameters A_c, R, V_d and I is explored around realistic values for theses parameters. For each batch, the acceleration amplitude at steady state is determined through a simulation run. A pump speed of 3000 rpm and a pump pressure of 450 bar define the steady state conditions. The choice for the steady state working point will be explained later on.

By varying I and V_d, while keeping A_c and R constant, the parameter space is two-dimensional. In **figure 5** the angular acceleration amplitude is presented within the parameter space. Each acceleration line connects all points in the **I**-V_d space that make the system respond with equal acceleration amplitudes. Parameters for which the resonance frequency of the swash system coincides with one of the Fourier orders of the swivel torque, have been connected by a bold line. The I-V_d space has been plotted four times, to cover the A_c and R dimensions of the parameter space. The actuator cup area is either 234 mm^2 or 468 mm^2, the actuator leverage is either 50 mm or 65 mm. An actuator set with A_c=234 mm^2 has cups which are equal in size as those used in the rotating group. The maximum stroke of these standard cups restricts the maximum leverage to 65 mm.

From the parameter variation it can be deduced that when designing the swash system, the values for A_c, R, V_d and I should be chosen with care. Near area's where resonance occurs, the acceleration amplitudes increase. For a specific barrel design, a critical acceleration level can be defined (see equation 1). Combining the critical acceleration level with figure 5, the parameter space can be divided into two parts: one for which the swash plate accelerations exceed the critical level, and one for which they remain below it. The swash system parameters must be chosen inside the acceptable region. Thus the swash system can be optimized with respect to the barrel design, pump speed range and maximum operating pressure.

What now can be expected from a similar analysis in other working points? For a given pump speed, the Fourier expansion of the swivel torque can be calculated. **Figure 6** shows the Fourier expansion for three speeds: 1000, 2000 and 3000 rpm. The four-dimensional parameter space comprises all realistic parameter values. Within this parameter set, two particular subsets exist; one for which the system's natural frequency is minimal ($f_{0,min}$), and one for which it reaches a maximum value ($f_{0,max}$). The corresponding frequency range has been drawn to figure 6. If an order of the excitation lies within the frequency range defined by these two extremes, a region of resonance exists within the parameter space.

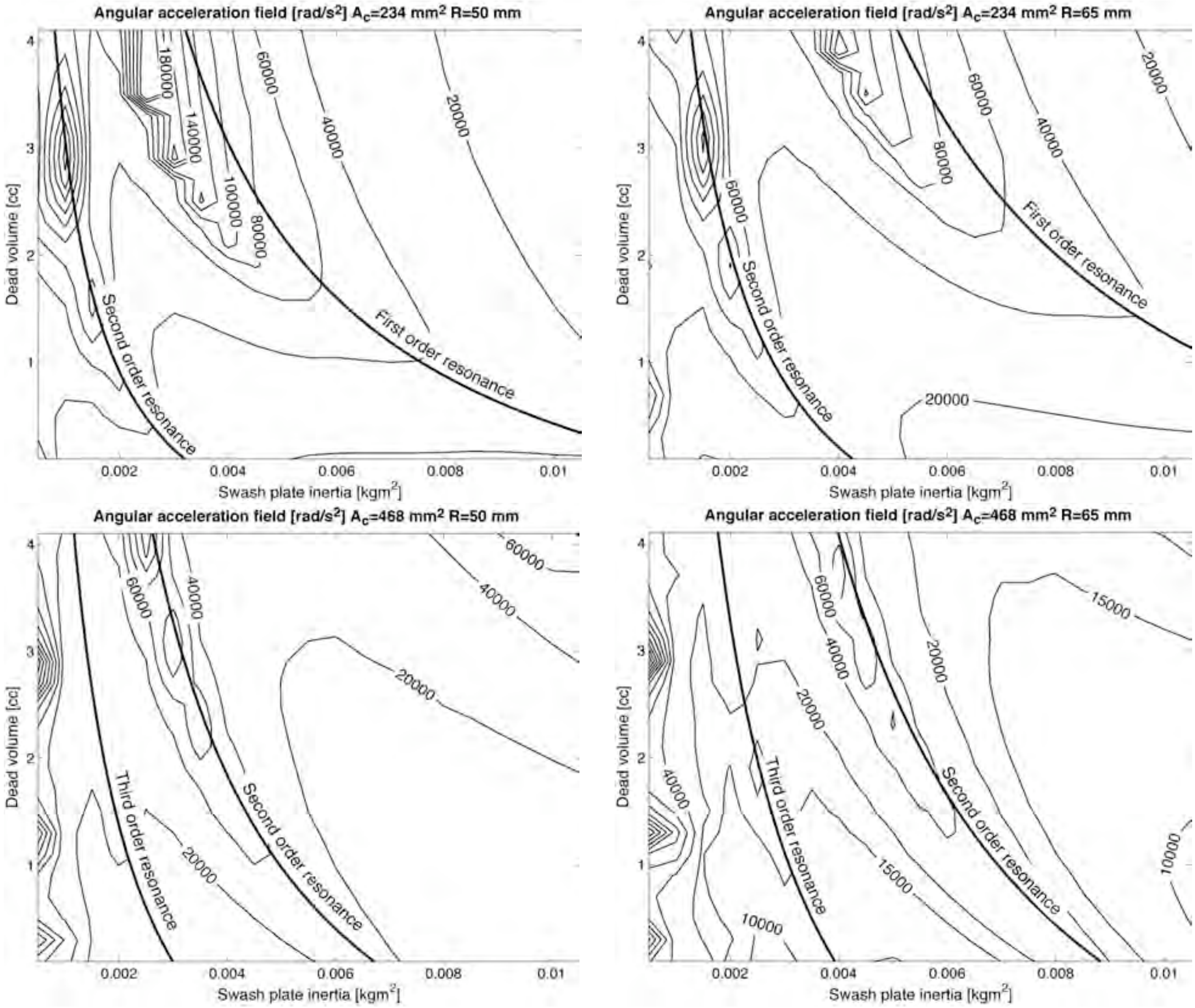


Figure 5 Parameter variation based on numerical simulations at 3000 rpm and 450 bar. The parameter space spanned by dead volume and inertia is depicted for different actuator areas and leverage. Each graph corresponds to a different combination of actuator area and leverage.

The pump speed is chosen such that at least the first three signal orders, which contain the bulk of the signal content, lie well within the frequency region. The maximum nominal speed of 3000 rpm satisfies that requirement. Lower speeds only influence the resonance region’s position fractionally. Pump pressure is set at the highest value, since the swivel torque signal then has a maximal amplitude.

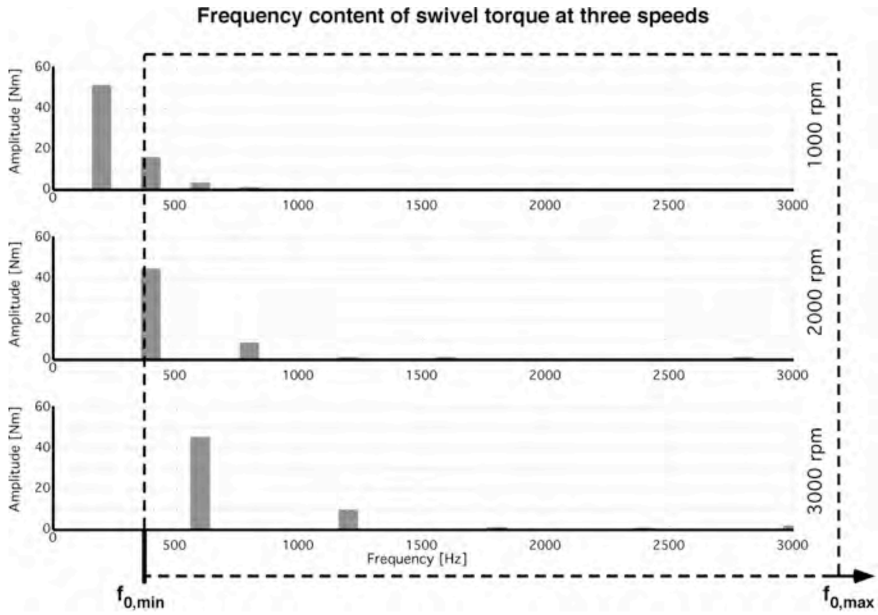


Figure 6 The frequency content of the swivel torque at three speeds: 1000, 2000 and 3000. The dashed rectangle represents the frequency range defined by the minimal and maximal natural frequency of the explored parameter space.

6 Conclusions and recommendations

For the recently presented Floating Cup pump with variable displacement, the design of the swash system deserves special attention. The main oil flow is fed through the swash plates to and from the pistons, making a tight interface between barrel and swash plate crucial. This paper discusses a systematic dynamical analysis that helps to understand the swash system’s dynamical behaviour. The angular acceleration of the swash plate around its axis turns out to be the main criterion for synchronised rotation of barrel and swash plate.

The governing design parameters have been identified and varied, to visualise their influence on the maximum occurring swash acceleration. With the analysis presented in this paper, subspaces of the total design parameter space can be identified, for which the maximum swash acceleration stays below an assumed maximum tolerable value.

For two reasons, it may seem to be appropriate to start building a more accurate model of the variable pump. Firstly, defining the acceptable acceleration level accurately is quite a challenge, since the barrel is partly hydrostatically supported. The simplified representation of the interaction between barrel and swash plate, resulting in the maximum barrel acceleration as given in equation 1, could be exchanged for a model that takes deformations and tribological phenomena into account. This would make an estimate of the maximum tolerable swash acceleration possible.

Furthermore, the swivel torque used here, originates from the model of a constant displacement machine. During pump operation, the swash angle is not a constant but changes periodically [5]. The amplitude of the variation depends on the amplitude of the swivel torque as well as the stiffness of the actuator system. The vibration will affect the pressure pulsations coming from the pumping pistons. This means that the swivel torque itself is affected by the varying swash angle. In a more detailed analysis of the swash acceleration, this difference can be taken into account. It is expected that the difference lies mainly in the distribution of the signal content over the different signal orders. To certify this, a complete model of a variable Floating Cup piston pump can be used.

Although the means for a more advanced simulation model are available, detailed simulation is a very time consuming activity. Moreover, it is not guaranteed that a more complex model, with a higher number of uncertain parameters, predicts the pump's behaviour correctly and helps in the actual understanding of the underlying physical phenomena. For that reason, the next step in the design process will be the manufacturing of a prototype for experimentation. The design exploration through experiments will be sustained by further elementary modelling and simulation.

References

1. Achten, P.A.J. *Designing the impossible pump*. in *Hydraulikdagarna*. 2003. Linköping.
2. Post,W. *Determination of steady-state performance of Innas Floating Cup type of axial piston pumps (ISO 4409)*. In *DCT-report 2004-127*. 2004. Eindhoven University of Technology.
3. Achten, P., Brink,T. van den,Schellekens, M. *Design of a variable displacement Floating Cup pump*. in *The ninth scandinavian international conference on fluid power*. 2005. Linköping, Sweden.
4. Manring, N.D., Johnson, R.E. *Swivel torque within a variable-displacement pump*. in *46th National Conference on Fluid Power*. 1994: National Fluid Power Association.
5. Nervegnia, N., Salvatore, M, Lettini, A., Guidetti, M., *Zweikreisverstellpumpe in axialkolbenbauart.* O+P Ölhydraulik und Pneumatik, 2004. **July**(7): p. 410-417.

Appendix

The swash system of an axial piston open circuit pump has been schematized in figure 2. The actuator with piston area A_c is mechanically connected to the swash plate inertia I at a given distance R from the swash axis. The swash angle is described by the angle φ which corresponds to the piston position x. The actuator is connected to the load pressure p_A and the low pressure p_T through the restrictions with opening diameters D_1 and D_2. The actuator pressure is the time varying state p(t).

Assuming turbulent flow, the flow through the restrictions is a nonlinear function of the pressure drop over each restriction. The total flow into the actuator is the result of the flows at the pressure junction.

$$Q_1 = Q_2 + Q_3 \Leftrightarrow Q_2 = Q_1 - Q_3 \qquad (2)$$

Depending on the junction pressure the resulting flow into the actuator varies as depicted in graph **figure 7**.

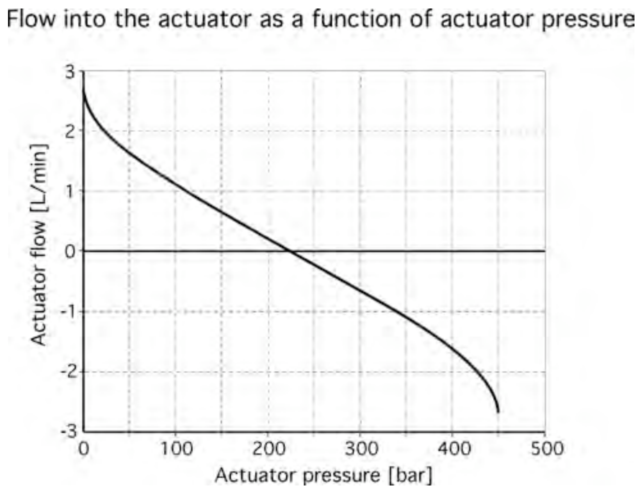


Figure 7 Relationship between actuator flow and pressure. Both restrictions are set at an opening diameter of 0.5 mm.

The pressure change $\dot{p}$ inside the actuator is caused on one side by oil flow into the actuator ($\dot{p}_{flow}$) and on the other by movement of the actuator piston ($\dot{p}_{piston}$). The pressure rise due to oil flow is governed by the actuator pressure in combination with the pump pressure p_A and the tank pressure p_T. In other words, the pressure change $\dot{p}_{flow}$ is a function of p. The movement of the piston is mechanically linked to the swash plate position. In other words, the pressure change $\dot{p}_{piston}$ is a function of the swash angle. The swash angle follows from the loads acting on the swash plate around the swash axis. There are three torques acting around the swash axis: the actuator torque M_{act}, the compensator torque M_{comp} and the swivel torque M_{swivel}.

Summarizing the above, the model behaviour of the swash system can be described by two differential equations concerning two states: the actuator pressure p(t) and the swash plate angle φ(t).

$$\begin{cases} \dot{p} = \dot{p}_{piston}(\varphi) + \dot{p}_{flow}(p) \\ \ddot{\varphi} = \frac{1}{I}\left(M_{act}(p) - M_{comp} + M_{swivel}\right) \end{cases} \tag{3}$$

The dynamic pressure variations inside the actuator are related to the actuator volume change through the oil compression modulus. The variation in time of the actuator pressure is linearly proportional to the actuator volume variation. Volume variation is caused by a change in piston position and by oil flow in or out of the actuator.

$$\dot{V} = -\frac{V_0}{E_{oil}}\dot{p} \tag{4}$$

$$\dot{V} = A_c R\dot{\varphi} - Q_2 \tag{5}$$

In equation 3, $\dot{p}_{flow}(p)$ is the only non-linear component. For better understanding of the physics behind the system dynamics, the non-linear function will be linearised. Figure 7 shows that linearisation around a steady state pressure p_{SS} is possible, as long as p_{SS} lies between 0 and 450 bar. After linearisation, the actuator torque can be split into two parts: a time varying part and a steady state part. The latter is a constant term, just like the compensator torque. For simplicity, the restriction diameters D_1 and D_2 are chosen to be equal. Choosing D_1 different from D_2 would only affect the leakage flow rate Q_3 to the tank.

$$\begin{cases} \dot{p} = -\dfrac{E_{oil}}{V_0} A_c R \dot{\varphi} + \dfrac{E_{oil}}{V_0} C_d \sqrt{\dfrac{2}{\rho_{oil}}} D_1{}^2 \dfrac{\pi}{4} \left(\sqrt{p_A - p} - \sqrt{p - p_T} \right) \\ \ddot{\varphi} = \dfrac{A_c R}{I} p - \dfrac{M_{comp}}{I} + \dfrac{M_{swivel}}{I} \end{cases} \tag{6}$$

Together with the swivel torque the constant loads can be gathered into one load term; the external load $M_{external}$. If the swash speed ω is used as a system state instead of the swash angle, the set of linearised differential equations describing the swash system behaviour becomes of first order and reads:

$$\begin{cases} \Delta\dot{p} = C_1 \omega + C_2 \Delta p \\ \dot{\omega} = C_4 \Delta p + \dfrac{1}{I} M_{external} \end{cases} \tag{7}$$

with

$$\Delta p = p - \frac{C_3}{C_2}$$

$$C_1 = -\frac{E_{oil}}{V_0} A_c R$$

$$C_2 = -\frac{E_{oil}}{V_0} C_d \sqrt{\frac{2}{\rho_{oil}}} D_1{}^2 \frac{\pi}{4} \left(\frac{1}{2\sqrt{(p_A - p_{ss})}} + \frac{1}{2\sqrt{p_{ss}}} \right)$$

$$C_3 = -\frac{E_{oil}}{V_0} C_d \sqrt{\frac{2}{\rho_{oil}}} D_1{}^2 \frac{\pi}{4} \left(\frac{2p_A - p_{ss}}{2\sqrt{(p_A - p_{ss})}} - \frac{p_{ss}}{2\sqrt{p_{ss}}} \right)$$

$$C_4 = \frac{A_c R}{I}$$

The pressure Δp stands for the pressure variations around the steady state pressure. The constants C_1, C_2, C_3 and C_4 are functions of the system parameters and the steady state pressure.

We are interested in the swash plate angular acceleration caused by the external load. One way to formulate the relationship is through the determination of the system 'gain'. If the external torque is seen as the system input and the angular acceleration as the output, the gain is the frequency dependent scaling factor between the two. In other words, if $M_{external}$ would be a sinusoidal function in time with amplitude M and frequency f, the system output would also be a sinusoidal function, since the model is linearised. The ratio between the output and the input amplitude is equal to the system gain, belonging to that frequency. Apart from that, the gain also depends on system parameters like swash plate inertia, actuator area etcetera. We are interested in those system parameters that influence the system gain the most. By tuning the governing parameters, the swash plate's angular acceleration can be controlled.

By making use of the Laplace transform of equation 7, the transfer function of the swash system becomes:

$$\begin{cases} sP(s) = C_1 \Omega(s) + C_2 P(s) \\ s\Omega(s) = C_4 P(s) + \dfrac{1}{I} M_{external}(s) \end{cases} \Rightarrow G^*(s) = \frac{\Omega(s)}{M_{external}(s)} = \frac{s - C_2}{I\left(s^2 - C_2 s - C_1 C_4\right)} \tag{8}$$

The capitalised functions of Laplace operator s, are the Laplace transforms of the corresponding time dependent functions $\Delta p(t)$, $\omega(t)$ and $M_{external}(t)$. The transfer function $G^*(s)$ gives the relationship between the external torque and the swash speed. To find the relationship between the external torque and the angular acceleration in the frequency domain, $G^*(s)$ is simply multiplied by s since this is the equivalent of differentiation in the time domain. The transfer function can be rewritten in terms of a spring-damper-mass system with natural frequency f_0 and damping ζ:

$$G(s) = \frac{A(s)}{M_{external}(s)} = \frac{s^2 + 2\zeta f_0 s}{I\left(s^2 + 2\zeta f_0 s + f_0^2\right)} \tag{9}$$

Here the natural frequency and damping in terms of the system parameters are:

$$\begin{aligned} f_0 &= A_c R \sqrt{\frac{E_{oil}}{I V_0}} \\ \zeta &= \frac{\pi C_d D_1^2}{8 A_c R} \sqrt{\frac{E_{oil} I}{2\rho_{oil} p_{SS} V_0 (p_A - p_{ss})}} \left(\sqrt{p_A - p_{ss}} + \sqrt{p_{ss}}\right) \end{aligned} \tag{10}$$

If either f_0 or I go to infinity, the transfer function approaches zero. Therefore, A_c, R, I and V_0 are parameters that can be used to minimize the swash acceleration. The influence of the damping ratio strongly depends on the excitation frequency. This is demonstrated in **figure 8** where the gain magnitude is plotted as a function of the restriction opening diameter. All other parameters are kept constant, so D_1 is a measure for the damping ratio.

The gain has been calculated for excitation at the resonance frequency, and for excitation at half the resonance frequency. When D_1 goes to infinity, the transfer function becomes equal to I^{-1}. When resonance occurs, $D_1 \rightarrow \infty$ gives the minimum gain magnitude. Outside the resonance region the minimum gain magnitude is achieved by closing the restrictions. As a consequence, D_1 cannot be used to minimize the swash acceleration under all working conditions. The restriction diameter is therefore set at 0.5 mm; a value for which the steady state flow into the tank equals 1.5 L/min.

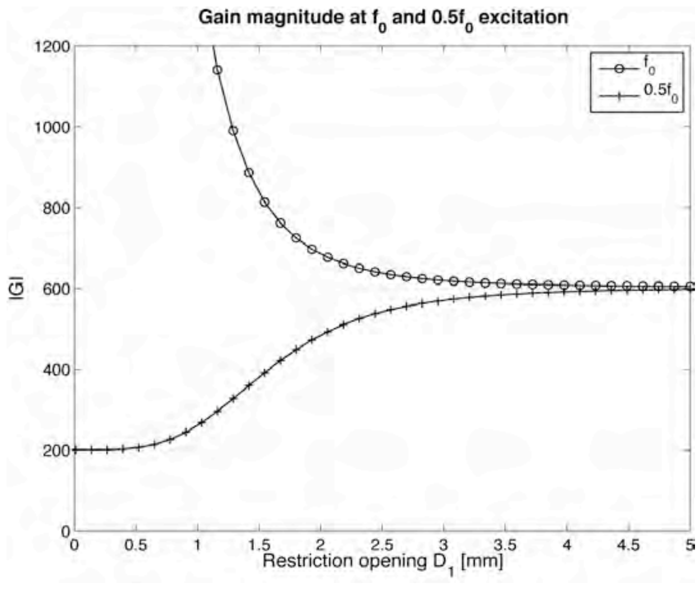


Figure 8 Gain magnitude for excitation at two different frequencies: f_0 and $0.5f_0$. The swash plate inertia $I=0.0017$ kgm^2.